Numerical problems

An algorithm is said to be *numerically unstable* if the algorithm is guaranteed to find a good approximation to an algebraic or analytic problem if real-numbers were used throughout the calculation (requiring us to store infinitely many digits), but where the potential for accumulated error making any approximation meaningless if fixed-precision floating-point arithmetic is used.

For example, solving a system of linear equations is numerically unstable, but using partial pivoting can significantly reduce the introduction of numerical errors.

Alternatively, calculating the inverse of a matrix is never numerically stable. There may be certain matrices for which calculating the inverse is straight-forward; for example, calculating the inverse of a diagonal matrix with no non-zero entries on the diagonal satisfies that property, but there is no algorithm that will even guarantee an inversion on even a tri-diagonal matrix (one with non-zero entries both in the super- and sub-diagonals as well as on the diagonal.

Calculating the determinant using the algorithms you unfortunately saw in first year is also slow and numerically unstable. The run time is $O(n!)$, while if one converts a matrix to row-echelon form, you can calculate the number

Classification of problems

The types of problems that we are attempting to solve may be summarized as follows: finding approximations to solutions to

1. expressions that have a fixed value,
2. algebraic equations or systems of algebraic equations,
3. analytic equations or systems of analytic equations, and
4. optimization problems.

We will quickly give an overview of each

Evaluating expressions

In this case, we may be trying to find the value of an integral, a derivative or values associated with matrices such as the norms of vectors or matrices or the determinant of a matrix.

Algebraic equations or systems of algebraic equations

An algebraic equation is an equation involving unknown variables. The objective is to approximate values of those variables that satisfy the equation or equations. For example, this may include finding solutions to systems of linear equations or finding roots of functions.

Analytic equations or systems of analytic equations

An analytic equation is generally an equation that contains an unknown function where the equation involves both integrals and derivatives of that equation. This may include differential equations in one variable or partial differential equations in two or more variables.

Optimization problems

An objective function is a real-valued function involving one or more variables. There may or may not be constraints on those variables. Optimizing such a function involves finding values of those variables that satisfy any constraints that also either minimize or maximize the value of the objective function.

Classification of errors

When approximating solutions to mathematical problems, there are many possible sources of error.